CP Violation from Finite Groups

Maximilian Fallbacher

in collaboration with M.-C. Chen, K. T. Mahanthappa, M. Ratz and A. Trautner

'CP Violation from Finite Groups', arXiv: 1402.0507 [hep-ph].

Technische Universität München

Department Physik

26 February 2014

Joint Particle Seminar, UC Irvine

CP violation and flavour physics

I

Direct evidence of CP violation in flavour observables.

$\Rightarrow$ Flavour and CP problems connected?

II

Powerful framework for flavour model building:

Finite symmetries.

Finite symmetries and flavour models

- Mixing parameters predicted by the (broken) symmetry.
- Can suppress flavour changing neutral currents.
- Non-abelian to yield non-trivial structures.
- No Goldstone bosons.
- Examples: $\mathrm{A}_4, \mathrm{T}', \mathrm{S}_4, \Delta(27), \ldots$

CP violation and flavour physics

I

Direct evidence of CP violation in flavour observables.

$\Rightarrow$ Flavour and CP problems connected?

II

Powerful framework for flavour model building:

Finite symmetries.

Investigate interplay between CP and finite symmetries.

Discussion not limited to flavour symmetries.

Outline

1. Motivation
2. Generalised CP and finite groups
3. Bickerstaff–Damhus automorphisms and bases with real CG`s
4. Three types of finite groups
5. Examples
6. Conclusion

Canonical CP transformation

- Single scalar field operator:

$$\phi(x) = \int \mathrm{d}^3 p \, \frac{1}{2E_{\vec{p}}} \left[\boldsymbol{a}(\vec{p}) \, \mathrm{e}^{-\mathrm{i}\, p\cdot x} + \boldsymbol{b}^\dagger(\vec{p}) \, \mathrm{e}^{\mathrm{i}\, p\cdot x} \right]$$

- Canonical CP transformation:

particle $\Leftrightarrow$ anti-particle

$$\vec{x} \Leftrightarrow -\vec{x}$$

$$\vec{p} \Leftrightarrow -\vec{p}$$

Canonical CP transformation

- Single scalar field operator:

$$\phi(x) = \int \mathrm{d}^3 p \, \frac{1}{2E_{\vec{p}}} \left[\boldsymbol{a}(\vec{p}) \, \mathrm{e}^{-\mathrm{i}\, p\cdot x} + \boldsymbol{b}^\dagger(\vec{p}) \, \mathrm{e}^{\mathrm{i}\, p\cdot x} \right]$$

- Canonical CP transformation:

$$(\boldsymbol{CP}) \, \boldsymbol{a}(\vec{p}) \, (\boldsymbol{CP})^{-1} = \eta_{\boldsymbol{CP}} \, \boldsymbol{b}(-\vec{p})$$
$$(\boldsymbol{CP}) \, \boldsymbol{b}(\vec{p}) \, (\boldsymbol{CP})^{-1} = \eta^*_{\boldsymbol{CP}} \, \boldsymbol{a}(-\vec{p})$$

$$\phi(t, \vec{x}) \xmapsto{\boldsymbol{CP}} \eta_{\boldsymbol{CP}} \, \phi(t, -\vec{x})^*$$

Inconsistency of CP and finite symmetries?

Holthausen, Lindner and Schmidt (2013)
Feruglio, Hagedorn and Ziegler (2013)

- Setting: A_4 with x, y in $\mathbf{3}$, ϕ in $\mathbf{1}_2$ and $\omega = \mathrm{e}^{\frac{2\pi \mathrm{i}}{3}}$.

$$\left[\phi_{\mathbf{1}_2} \otimes (x_{\mathbf{3}} \otimes y_{\mathbf{3}})_{\mathbf{1}_1}\right]_{\mathbf{1}_0} \propto \phi \left(x_1\, y_1 + \omega^2\, x_2\, y_2 + \omega\, x_3\, y_3\right)$$

- Canonical $\boldsymbol{CP}$:

$$\left[\phi_{\mathbf{1}_2} \otimes (x_{\mathbf{3}} \otimes y_{\mathbf{3}})_{\mathbf{1}_1}\right]_{\mathbf{1}_0} \xmapsto{\boldsymbol{CP}} \phi^* \left(x_1^*\, y_1^* + \omega^2\, x_2^*\, y_2^* + \omega\, x_3^*\, y_3^*\right)$$

Not A_4 invariant.

- Reason: complex Clebsch–Gordan coefficients.

Generalised CP transformations

Ecker, Grimus and Konetschny (1981)
Branco, Gerard and Grimus (1984)

- Particles and anti-particles are multiplets:

$$\boldsymbol{a} = (\boldsymbol{a}_1, \ldots, \boldsymbol{a}_n),\ \boldsymbol{b} = (\boldsymbol{b}_1, \ldots, \boldsymbol{b}_n).$$

- Generalised CP transformation with unitary U_{CP}:

$$(\boldsymbol{CP})\, \boldsymbol{a}(\vec{p})\, (\boldsymbol{CP})^{-1} = U_{CP}\, \boldsymbol{b}(-\vec{p})$$
$$(\boldsymbol{CP})\, \boldsymbol{b}(\vec{p})\, (\boldsymbol{CP})^{-1} = \boldsymbol{a}(-\vec{p})\, {U_{CP}}^{\dagger}$$

$$\Phi(t, \vec{x}) \xmapsto{\widetilde{\boldsymbol{CP}}} U_{CP}\, \Phi(t, -\vec{x})^*$$

Inconsistency of CP and finite symmetries?

Holthausen, Lindner and Schmidt (2013)
Feruglio, Hagedorn and Ziegler (2013)

- Setting: A_4 with x, y in $\mathbf{3}$, ϕ in $\mathbf{1}_2$ and $\omega = \mathrm{e}^{\frac{2\pi\mathrm{i}}{3}}$.

$$\left[\phi_{\mathbf{1}_2} \otimes (x_{\mathbf{3}} \otimes y_{\mathbf{3}})_{\mathbf{1}_1}\right]_{\mathbf{1}_0} \propto \phi \left(x_1\, y_1 + \omega^2\, x_2\, y_2 + \omega\, x_3\, y_3\right)$$

- Canonical CP:

$$\left[\phi_{\mathbf{1}_2} \otimes (x_{\mathbf{3}} \otimes y_{\mathbf{3}})_{\mathbf{1}_1}\right]_{\mathbf{1}_0} \xmapsto{CP} \phi^* \left(x_1^*\, y_1^* + \omega^2\, x_2^*\, y_2^* + \omega\, x_3^*\, y_3^*\right)$$

Not A_4 invariant.

- Reason: complex Clebsch–Gordan coefficients.
- Generalised CP: Exchange second and third components: $2 \leftrightarrow 3$.

$$\left[\phi_{\mathbf{1}_2} \otimes (x_{\mathbf{3}} \otimes y_{\mathbf{3}})_{\mathbf{1}_1}\right]_{\mathbf{1}_0} \xmapsto{\widetilde{CP}} \phi^* \left(x_1^*\, y_1^* + \omega^2\, x_3^*\, y_3^* + \omega\, x_2^*\, y_2^*\right)$$

Generalised CP transformations

Holthausen, Lindner and Schmidt (2013)
Feruglio, Hagedorn and Ziegler (2013)

Generalised CP

$$\Phi(t,\vec{x}) \xmapsto{\widetilde{CP}} U_{CP}\,\Phi(t,-\vec{x})^*$$

Finite group G

$$\Phi(x) \xmapsto{G} \rho(g)\,\Phi(x)$$

Consistency condition

Consistent if and only if
there is an automorphism u of G such that

$$\rho(u(g)) = U_{CP}\,\rho(g)^*\,U_{CP}{}^{-1}\,, \qquad \forall\, g \in G.$$

Does every automorphism yield a CP transformation?

Constraints on generalised CP

Generalised CP?

$$\rho(u(g)) = U_{CP}\,\rho(g)^*\,U_{CP}^{-1}\,, \qquad \forall\, g \in G$$

$$\Phi(t,\vec{x}) \xmapsto{\widetilde{CP}} U_{CP}\,\Phi(t,-\vec{x})^*$$

- Holthausen, Lindner and Schmidt (2013):
  Φ contains all (scalar) fields of the model and their conjugates.

$$\Phi = (\phi^1,\dots,\phi^n,(\phi^1)^*,\dots,(\phi^n)^*)\,, \qquad \phi^i \in \boldsymbol{R}_k\,.$$

Constraints on generalised CP

Generalised CP?

$$\rho(u(g)) = U_{CP}\, \rho(g)^*\, U_{CP}^{-1}\,, \qquad \forall\, g \in G$$

$$\Phi(t,\vec{x}) \xrightarrow{\widetilde{CP}} U_{CP}\, \Phi(t,-\vec{x})^*$$

- Possible transformations from solutions of consistency condition ($i \neq j$):

$$\text{(i)}\quad \phi^i \mapsto U\,(\phi^i)^*\,, \qquad \text{(iii)}\quad \phi^i \mapsto U\,(\phi^j)^*\,,$$

$$\text{(ii)}\quad \phi^i \mapsto U\,\phi^i\,, \qquad \text{(iv)}\quad \phi^i \mapsto U\,\phi^j\,.$$

- Independence of particle content of a model.

- Connection to observed CP violation and baryogenesis:
  $\Rightarrow$ CP should invert the quantum numbers of a field.

Constraints on generalised CP

Generalised CP?

$$\rho(u(g)) = U_{CP}\, \rho(g)^*\, {U_{CP}}^{-1}\,, \qquad \forall\, g \in G$$

$$\Phi(t,\vec{x}) \xmapsto{\widetilde{CP}} U_{CP}\, \Phi(t,-\vec{x})^*$$

- Possible transformations from solutions of consistency condition ($i \neq j$):

$$\text{(i)}\;\; \phi^i \mapsto U\,(\phi^i)^*\,, \qquad \text{(iii)}\;\; \phi^i \mapsto U\,(\phi^j)^*\,,$$

$$\text{(ii)}\;\; \cancel{\phi^i \mapsto U\,\phi^i}\,, \qquad \text{(iv)}\;\; \phi^i \mapsto U\,\phi^j\,.$$

- Independence of particle content of a model.
- Connection to observed CP violation and baryogenesis:
  $\Rightarrow$ CP should invert the quantum numbers of a field.

Constraints on generalised CP

Generalised CP?

$$\rho(u(g)) = U_{CP}\,\rho(g)^*\,U_{CP}{}^{-1}\,, \qquad \forall\, g \in G$$

$$\Phi(t,\vec{x}) \xmapsto{\widetilde{CP}} U_{CP}\,\Phi(t,-\vec{x})^*$$

- Possible transformations from solutions of consistency condition ($i \neq j$):

$$\text{(i)}\quad \phi^i \mapsto U\,(\phi^i)^*\,, \qquad \text{(iii)}\quad \cancel{\phi^i \mapsto U\,(\phi^j)^*}\,,$$

$$\text{(ii)}\quad \cancel{\phi^i \mapsto U\,\phi^i}\,, \qquad \text{(iv)}\quad \phi^i \mapsto U\,\phi^j\,.$$

- Independence of particle content of a model.
- Connection to observed CP violation and baryogenesis:
  $\Rightarrow$ CP should invert the quantum numbers of a field.

Constraints on generalised CP

Generalised CP?

$$\rho(u(g)) = U_{CP}\,\rho(g)^*\,{U_{CP}}^{-1}\,, \qquad \forall\, g \in G$$

$$\Phi(t,\vec{x}) \xmapsto{\widetilde{CP}} U_{CP}\,\Phi(t,-\vec{x})^*$$

- Possible transformations from solutions of consistency condition ($i \neq j$):

  (i) $\phi^i \mapsto U\,(\phi^i)^*$, (iii) ~~$\phi^i \mapsto U\,(\phi^j)^*$~~,

  (ii) ~~$\phi^i \mapsto U\,\phi^i$~~, (iv) ~~$\phi^i \mapsto U\,\phi^j$~~.

- Independence of particle content of a model.
- Connection to observed CP violation and baryogenesis:
  $\Rightarrow$ CP should invert the quantum numbers of a field.

Class-inverting

- Demand that for each field ϕ_i in $\boldsymbol{R}_i$,

$$\phi_i(t,\vec{x}) \xrightarrow{\widetilde{\boldsymbol{CP}}} U_i\,\phi_i(t,-\vec{x})^*\,, \qquad U_i = U(\boldsymbol{R}_i).$$

$\Rightarrow$ $\boldsymbol{CP}$ transformation independent of the particle content.

$\Rightarrow$ Inverts quantum numbers.

Class-inverting

$$\Phi = \left(\begin{array}{c} \uparrow \\ \phi_{i_1} \\ \downarrow \\ \hline \uparrow \\ \phi_{i_2} \\ \downarrow \\ \hline \vdots \end{array}\right) \xmapsto{\widetilde{CP}} \left(\begin{array}{c|c|c} \nwarrow \quad \nearrow & & \\ U_{i_1} & & \\ \swarrow \quad \searrow & & \\ \hline & \nwarrow \quad \nearrow & \\ & U_{i_2} & \\ & \swarrow \quad \searrow & \\ \hline & & \ddots \end{array}\right) \left(\begin{array}{c} \uparrow \\ \phi^*_{i_1} \\ \downarrow \\ \hline \uparrow \\ \phi^*_{i_2} \\ \downarrow \\ \hline \vdots \end{array}\right) = U_{CP}\,\Phi^* .$$

Class-inverting

- Demand that for each field ϕ_i in $\boldsymbol{R}_i$,

$$\phi_i(t, \vec{x}) \xmapsto{\widetilde{CP}} U_i\, \phi_i(t, -\vec{x})^*\,, \qquad U_i = U(\boldsymbol{R}_i).$$

- New consistency condition for each $\boldsymbol{R}_i$,

$$\rho_i(u(g)) = U_i\, \rho_i(g)^*\, U_i^{-1}\,, \qquad \forall\, g \in G\,, \qquad \forall\, i.$$

- Has a solution for all $\boldsymbol{R}_i$ if and only if u is class-inverting

$\Leftrightarrow$ $u(g)$ and g^{-1} in the same conjugacy class for all $g \in G$.

Summary of generalised CP transformations

Generalised CP

$$\phi_i(t,\vec{x}) \xmapsto{\widetilde{\boldsymbol{CP}}} U_i\,\phi_i(t,-\vec{x})^*\,, \qquad U_i = U(\boldsymbol{R}_i)$$

$$\rho_i(u(g)) = U_i\,\rho_i(g)^*\,U_i^{-1}\,, \qquad \forall\, g\in G\,, \qquad \forall\, i$$

u has to be class-inverting.

Real or complex Clebsch–Gordan coefficients

How is this connected to complex Clebsch–Gordan coefficients?

The Bickerstaff–Damhus automorphism

Bickerstaff and Damhus (1985)

Existence of an automorphism u such that

$$\rho_i(u(g)) = \rho_i(g)^*\,, \qquad \forall\, g \in G\,,\ \forall\, i\,.$$

$\Updownarrow$

Chosen basis has real Clebsch–Gordan coefficients.

Basis-dependent statement

The Bickerstaff–Damhus automorphism

Bickerstaff and Damhus (1985)

Existence of an automorphism u such that

$$\rho_i(u(g)) = U_i\, \rho_i(g)^*\, U_i^{-1}\,, \quad U_i \text{ symmetric,} \qquad \forall\, g \in G\,,\ \forall\, i\,.$$

$\Updownarrow$

Existence of a basis with real Clebsch–Gordan coefficients.

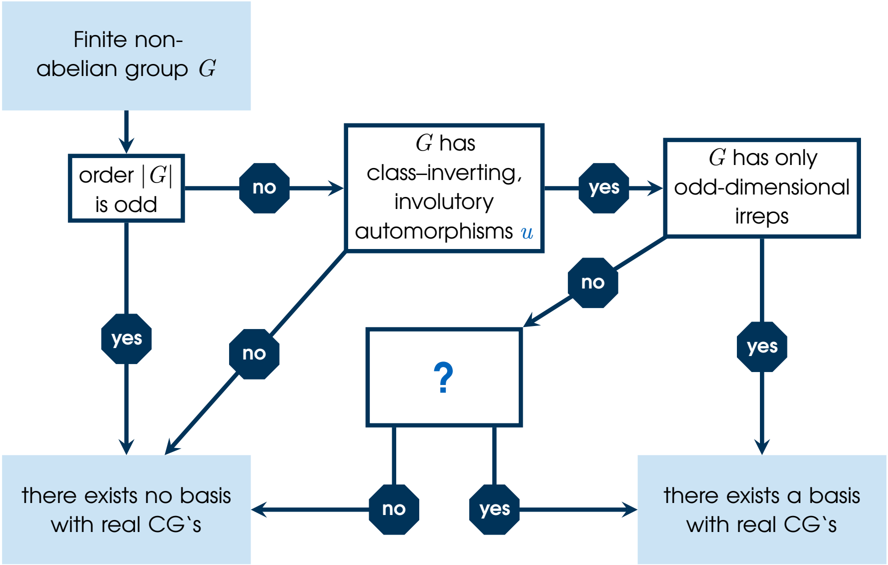
Basis-independent statement

u automatically class-inverting and involutory

$U_i = \mathbb{1}$ is the basis with real CG`s

$\Rightarrow$ generalised $\boldsymbol{CP}$ = canonical $\boldsymbol{CP}$

Real Clebsch–Gordan coefficients



The twisted Frobenius–Schur indicator

- The Frobenius–Schur indicator:

$$\mathrm{FS}(\boldsymbol{R}_i) = \frac{1}{|G|} \sum_{g\in G} \chi_i(g^2) = \frac{1}{|G|} \sum_{g\in G} \mathrm{tr}\left[\rho_i(g)^2\right]$$

$$\mathrm{FS}(\boldsymbol{R}_i) = \begin{cases} +1, & \text{if } \boldsymbol{R}_i \text{ is a real representation,} \\ 0, & \text{if } \boldsymbol{R}_i \text{ is a complex representation,} \\ -1, & \text{if } \boldsymbol{R}_i \text{ is a pseudo-real representation.} \end{cases}$$

The twisted Frobenius–Schur indicator

- The Frobenius–Schur indicator:

$$\mathrm{FS}(\boldsymbol{R}_i) = \frac{1}{|G|} \sum_{g \in G} \chi_i(g^2) = \frac{1}{|G|} \sum_{g \in G} \mathrm{tr}\left[\rho_i(g)^2\right]$$

- The twisted Frobenius–Schur indicator:

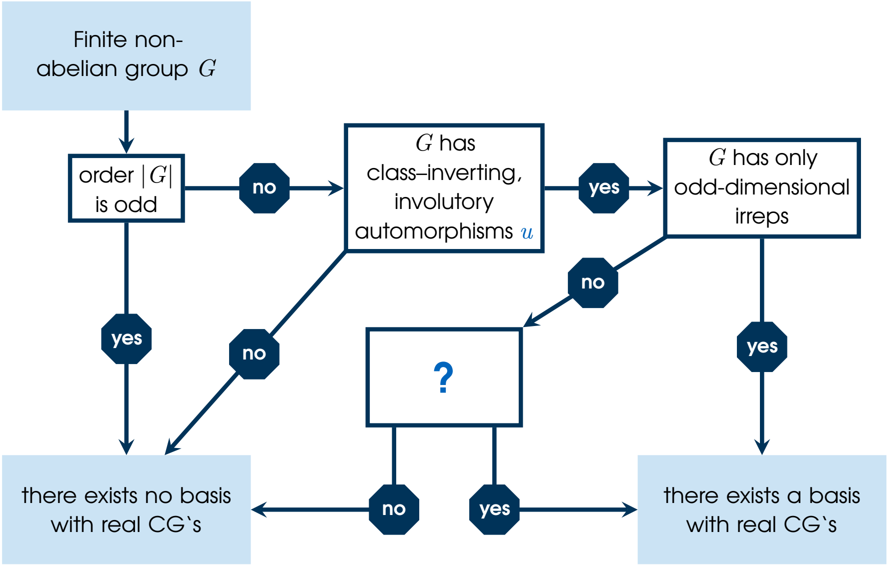
Kawanaka and Matsuyama (1990)

$$\mathrm{FS}_u(\boldsymbol{R}_i) = \frac{1}{|G|} \sum_{g \in G} \chi_i(g\, u(g)) = \frac{1}{|G|} \sum_{g \in G} \mathrm{tr}\left[\rho_i(g)\, \rho_i(u(g))\right]$$

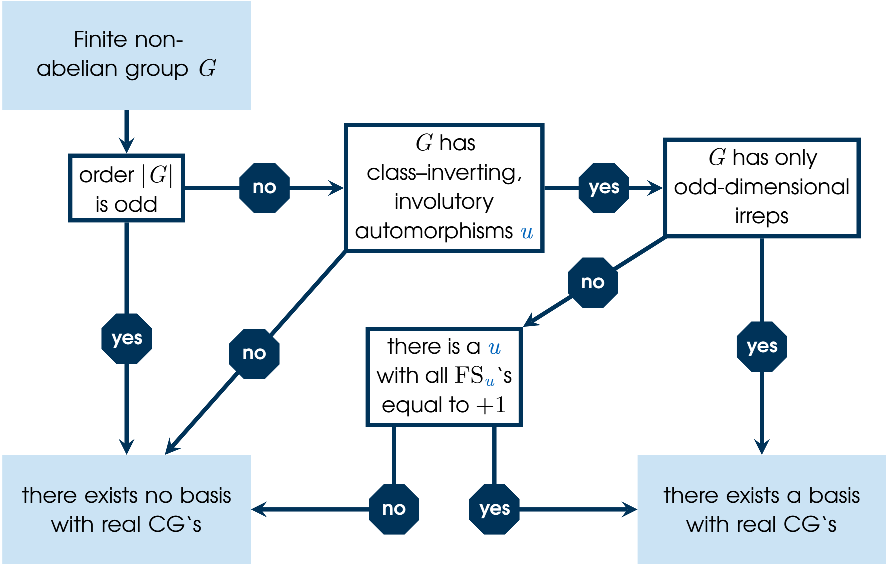
$$\mathrm{FS}_u(\boldsymbol{R}_i) = \begin{cases} +1 \quad \forall\, i, & \text{if } u \text{ is a BDA,} \\ \pm 1 \quad \forall\, i, & \text{if } u \text{ is class-inverting and involutory,} \\ \text{not only } \pm 1, & \text{otherwise.} \end{cases}$$

GAP – Groups, Algorithms, and Programming (2014)

Real Clebsch–Gordan coefficients



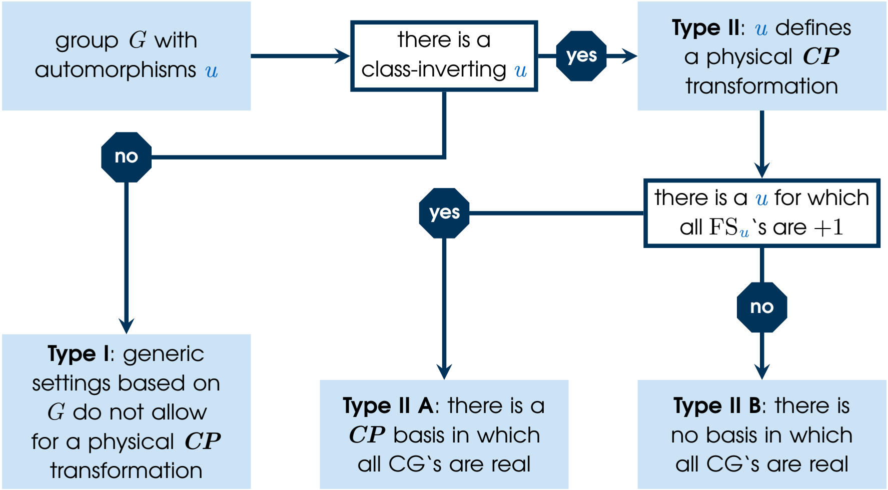
Real Clebsch–Gordan coefficients



Three types of finite groups

Does a finite group G have a proper $\boldsymbol{CP}$ transformation?

Three types of finite groups



Examples for the three types of finite groups

- **Type I:**

G	$\mathbb{Z}_5 \rtimes \mathbb{Z}_4$	T_7	$\Delta(27)$	$\mathbb{Z}_9 \rtimes \mathbb{Z}_3$
SG	(20,3)	(21,1)	(27,3)	(27,4)

$+$ (almost?) all odd order, non-abelian groups

- **Type II A:**

G	Q_8	A_4	$\mathbb{Z}_3 \rtimes \mathbb{Z}_8$	T'	S_4	A_5
SG	(8,4)	(12,3)	(24,1)	(24,3)	(24,12)	(60,5)

- **Type II B:**

G	$\Sigma(72)$	$((\mathbb{Z}_3 \times \mathbb{Z}_3) \rtimes \mathbb{Z}_4) \rtimes \mathbb{Z}_4$
SG	(72,41)	(144,120)

Example for type II A: T'

Type II A

There is a basis with real Clebsch–Gordan coefficients

Ishimori, Kobayashi, Ohki, Okada, Shimizu and Tanimoto (2010)

Example for type II A: T'

- Presentation for T'

$$S^4 = T^3 = (S\,T)^3 = e\,.$$

- Seven irreducible representations

$$\mathbf{1}_0\,,\ \mathbf{1}_1\,,\ \mathbf{1}_2\,,\ \mathbf{2}_0\,,\ \mathbf{2}_1\,,\ \mathbf{2}_2 \text{ and } \mathbf{3}\,.$$

- Bickerstaff–Damhus automorphism $u\,:\ (S,\,T) \mapsto (S^3,\,T^2)$

$$\mathbf{1}_i \xmapsto{u} \mathbf{1}_i{}^*\,, \qquad \mathbf{2}_i \xmapsto{u} \mathbf{2}_i{}^*\,, \qquad \mathbf{3} \xmapsto{u} \mathbf{3}^*\,.$$

- Twisted Frobenius–Schur indicators of u:

$\boldsymbol{R}$	$\mathbf{1}_0$	$\mathbf{1}_1$	$\mathbf{1}_2$	$\mathbf{2}_0$	$\mathbf{2}_1$	$\mathbf{2}_2$	$\mathbf{3}$
$\mathrm{FS}_u(\boldsymbol{R})$	1	1	1	1	1	1	1

Basis-change and phases

- Generalised CP transformation

$$\mathbf{1}_i \xmapsto{\widetilde{CP}} \mathbf{1}_i{}^* \,,$$
$$\mathbf{2}_i \xmapsto{\widetilde{CP}} \mathbf{2}_i{}^* \,,$$
$$\mathbf{3} \xmapsto{\widetilde{CP}} \mathbf{3}^* \,.$$

Type II A: CP constrains phases of couplings.

Basis-change and phases

- Generalised CP transformation

$$\mathbf{1}_i \xmapsto{\widetilde{CP}} (V_{\mathbf{1}_i}\, V_{\mathbf{1}_i}^T)\, \mathbf{1}_i{}^* \,,$$

$$\mathbf{2}_i \xmapsto{\widetilde{CP}} (V_{\mathbf{2}_i}\, V_{\mathbf{2}_i}^T)\, \mathbf{2}_i{}^* \,,$$

$$\mathbf{3} \xmapsto{\widetilde{CP}} (V_{\mathbf{3}}\, V_{\mathbf{3}}^T)\, \mathbf{3}^* \,.$$

Basis-change

$$\rho'_{\boldsymbol{R}_i}(g) = V_{\boldsymbol{R}_i}\, \rho_{\boldsymbol{R}_i}(g)\, V_{\boldsymbol{R}_i}^{-1}$$

Type II A: CP constrains phases of couplings.

Basis-change and phases

- Generalised CP transformation

$$\mathbf{1}_i \xmapsto{\widetilde{CP}} \mathrm{e}^{\mathrm{i}\alpha(\phi)} \left(V_{\mathbf{1}_i} \, V_{\mathbf{1}_i}^T \right) \mathbf{1}_i{}^* \,,$$

$$\mathbf{2}_i \xmapsto{\widetilde{CP}} \mathrm{e}^{\mathrm{i}\alpha(\phi)} \left(V_{\mathbf{2}_i} \, V_{\mathbf{2}_i}^T \right) \mathbf{2}_i{}^* \,,$$

$$\mathbf{3} \xmapsto{\widetilde{CP}} \mathrm{e}^{\mathrm{i}\alpha(\phi)} \left(V_{\mathbf{3}} \, V_{\mathbf{3}}^T \right) \mathbf{3}^* \,.$$

Re-phasing freedom of fields

$$\phi \to \mathrm{e}^{\mathrm{i}\alpha(\phi)} \, \phi$$

Type II A: CP constrains phases of couplings.

Example for type II B: $\Sigma(72)$

Type II B

No basis with real Clebsch–Gordan coefficients

Class-inverting automorphism $=$ physical $\boldsymbol{CP}$ transformation

Example for type II B: $\Sigma(72)$

- Six irreducible representations

$$\mathbf{1}_0\,,\ \mathbf{1}_1\,,\ \mathbf{1}_2\,,\ \mathbf{1}_3\,,\ \mathbf{2} \text{ and } \mathbf{8}\,.$$

- Ambivalent
  $\Leftrightarrow$ g and g^{-1} in the same conjugacy class for all g
  $\Leftrightarrow$ identity automorphism id is class-inverting

- (Twisted) Frobenius–Schur indicators of id:

$\boldsymbol{R}$	$\mathbf{1}_0$	$\mathbf{1}_1$	$\mathbf{1}_2$	$\mathbf{1}_3$	$\mathbf{2}$	$\mathbf{8}$
$\mathrm{FS}_{\mathrm{id}}(\boldsymbol{R})$	1	1	1	1	-1	1

Additional $\mathbb{Z}_2$

- Generalised CP transformation

$$\mathbf{1}_i \xmapsto{\widetilde{CP}} \mathbf{1}_i{}^* \,, \qquad \mathbf{2} \xmapsto{\widetilde{CP}} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \mathbf{2}^* \,, \qquad \mathbf{8} \xmapsto{\widetilde{CP}} \mathbf{8}^* \,.$$

- Perform $\widetilde{CP}$ twice: $\phi_i \xmapsto{\widetilde{CP}} U_i\, \phi_i^* \xmapsto{\widetilde{CP}} U_i\, U_i{}^*\, \phi_i = V_i\, \phi_i$,

$$V_{\boldsymbol{R}\neq\mathbf{2}} = \mathbb{1} \,, \qquad V_{\mathbf{2}} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}^* = - \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} .$$

$\Rightarrow$ Enlarges group by a $\mathbb{Z}_2$

- CP forbids terms like

$$\mathcal{L} \supset g\, (\mathbf{2} \otimes (\mathbf{8} \otimes \mathbf{8})_{\mathbf{2}})_{\mathbf{1}_0} \,.$$

Type II B: CP can forbid couplings.

Example for type I: $\Delta(27)$

Type I

No class-inverting automorphism

$=$

No physical CP transformation in generic settings

Example for type I: $\Delta(27)$

- Eleven irreducible representations

$$\mathbf{1}_0 \text{ to } \mathbf{1}_8\,,\ \mathbf{3} \text{ and } \mathbf{3}^*\,.$$

	S	X	Y	Ψ		Σ
$\Delta(27)$	$\mathbf{1}_0$	$\mathbf{1}_1$	$\mathbf{1}_3$	$\mathbf{3}$		$\mathbf{3}$
$\mathrm{U}(1)$	$q_\Psi - q_\Sigma$	$q_\Psi - q_\Sigma$	0	q_Ψ	$\neq$	q_Σ

$$\mathcal{L} \supset g_S \left[S_{\mathbf{1}_0} \otimes \left(\overline{\Psi} \otimes \Sigma\right)_{\mathbf{1}_0}\right]_{\mathbf{1}_0} + g_X \left[X_{\mathbf{1}_1} \otimes \left(\overline{\Psi} \otimes \Sigma\right)_{\mathbf{1}_2}\right]_{\mathbf{1}_0} + \\ + h_\Psi \left[Y_{\mathbf{1}_3} \otimes \left(\overline{\Psi} \otimes \Psi\right)_{\mathbf{1}_6}\right]_{\mathbf{1}_0} + h_\Sigma \left[Y_{\mathbf{1}_3} \otimes \left(\overline{\Sigma} \otimes \Sigma\right)_{\mathbf{1}_6}\right]_{\mathbf{1}_0} + \mathrm{h.\,c.}$$

Example for type I: $\Delta(27)$

- Eleven irreducible representations

$$\mathbf{1}_0 \text{ to } \mathbf{1}_8\,,\ \mathbf{3} \text{ and } \mathbf{3}^*\,.$$

	S	X	Y	Ψ		Σ
$\Delta(27)$	$\mathbf{1}_0$	$\mathbf{1}_1$	$\mathbf{1}_3$	$\mathbf{3}$		$\mathbf{3}$
$\mathrm{U}(1)$	$q_\Psi - q_\Sigma$	$q_\Psi - q_\Sigma$	0	q_Ψ	$\neq$	q_Σ

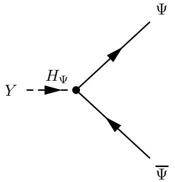
$$\mathcal{L} \supset (G_S)^{ij}\, S\,\overline{\Psi}_i\Sigma_j + (G_X)^{ij}\, X\,\overline{\Psi}_i\Sigma_j + \\ + (H_\Psi)^{ij}\, Y\,\overline{\Psi}_i\Psi_j + (H_\Sigma)^{ij}\, Y\,\overline{\Sigma}_i\Sigma_j + \mathrm{h.\,c.}$$

Y decay

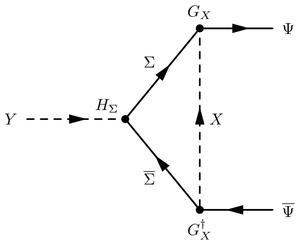
CP asymmetry

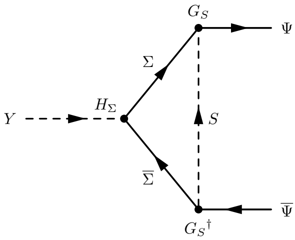
$$\varepsilon_{Y\to\overline{\Psi}\Psi} = \frac{\Gamma(Y \to \overline{\Psi}\Psi) - \Gamma(Y^* \to \overline{\Psi}\Psi)}{\Gamma(Y \to \overline{\Psi}\Psi) + \Gamma(Y^* \to \overline{\Psi}\Psi)} \,.$$

- Tree-level:



- Loop-level:





Y decay

CP asymmetry

$$\varepsilon_{Y\to\overline{\Psi}\Psi} = |g_S|^2 \ \mathrm{Im}\,(I_S)\ \mathrm{Im}\,(h_\Psi\, h_\Sigma^*) + |g_X|^2 \ \mathrm{Im}\,(I_X)\ \mathrm{Im}\,(\omega\, h_\Psi\, h_\Sigma^*)\,.$$

- $I_{S/X}$ are loop integrals, $\omega = \mathrm{e}^{\frac{2\pi \mathrm{i}}{3}}$.
- Invariant under re-phasing of fields.
- Two possibilities for vanishing asymmetry:

1

- $\mathrm{Im}\,(I_S) = \mathrm{Im}\,(I_X)$, requires $M_S = M_X$, and
- $|g_S| = |g_X|$ and
- $\arg\,(h_\Psi\, h_\Sigma^*) = -2\pi/6$.

2

- $\mathrm{Im}\,(I_S) \neq \mathrm{Im}\,(I_X)$ and
- $\arg\,(h_\Psi\, h_\Sigma^*)$ adjusted accordingly.

Y decay

CP asymmetry

$$\varepsilon_{Y\to\overline{\Psi}\Psi} = |g_S|^2 \,\mathrm{Im}\,(I_S)\,\mathrm{Im}\,(h_\Psi\, h_\Sigma^*) + |g_X|^2 \,\mathrm{Im}\,(I_X)\,\mathrm{Im}\,(\omega\, h_\Psi\, h_\Sigma^*)\,.$$

- $I_{S/X}$ are loop integrals, $\omega = \mathrm{e}^{\frac{2\pi \mathrm{i}}{3}}$.
- Invariant under re-phasing of fields.
- Two possibilities for vanishing asymmetry:

1

- $\mathrm{Im}\,(I_S) = \mathrm{Im}\,(I_X)$, requires $M_S = M_X$, and
- $|g_S| = |g_X|$ and
- $\arg\,(h_\Psi\, h_\Sigma^*) = -2\pi/6$.

2

- $\mathrm{Im}\,(I_S) \neq \mathrm{Im}\,(I_X)$ and
- $\arg\,(h_\Psi\, h_\Sigma^*)$ adjusted accordingly.

Y decay

CP asymmetry

$$\varepsilon_{Y \to \overline{\Psi}\Psi} = |g_S|^2 \, \mathrm{Im}\,(I_S)\, \mathrm{Im}\,(h_\Psi\, h_\Sigma^*) + |g_X|^2 \, \mathrm{Im}\,(I_X)\, \mathrm{Im}\,(\omega\, h_\Psi\, h_\Sigma^*)\,.$$

- $I_{S/X}$ are loop integrals, $\omega = \mathrm{e}^{\frac{2\pi \mathrm{i}}{3}}$.
- Invariant under re-phasing of fields.
- Two possibilities for vanishing asymmetry:

1

- $\mathrm{Im}\,(I_S) = \mathrm{Im}\,(I_X)$, requires $M_S = M_X$, and
- $|g_S| = |g_X|$ and
- $\arg\,(h_\Psi\, h_\Sigma^*) = -2\pi/6$.

2

- $\mathrm{Im}\,(I_S) \neq \mathrm{Im}\,(I_X)$ and
- $\arg\,(h_\Psi\, h_\Sigma^*)$ adjusted accordingly.

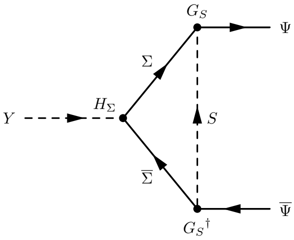
The model violates CP.

Amended toy model

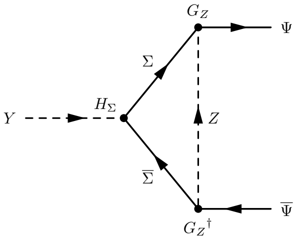
- Replace S (in $\mathbf{1}_0$) by Z in $\mathbf{1}_8$:

$$\mathcal{L} \supset g_Z \left[Z_{\mathbf{1}_8} \otimes \left(\overline{\Psi} \otimes \Sigma \right)_{\mathbf{1}_4} \right]_{\mathbf{1}_0} + g_X \left[X_{\mathbf{1}_1} \otimes \left(\overline{\Psi} \otimes \Sigma \right)_{\mathbf{1}_2} \right]_{\mathbf{1}_0} + \\ + h_\Psi \left[Y_{\mathbf{1}_3} \otimes \left(\overline{\Psi} \otimes \Psi \right)_{\mathbf{1}_6} \right]_{\mathbf{1}_0} + h_\Sigma \left[Y_{\mathbf{1}_3} \otimes \left(\overline{\Sigma} \otimes \Sigma \right)_{\mathbf{1}_6} \right]_{\mathbf{1}_0} + \mathrm{h.\,c.}$$

- Replace:



$\rightarrow$



Y decay in the amended model

CP asymmetry

$$\varepsilon_{Y \to \overline{\Psi}\Psi} = |g_X|^2 \, \mathrm{Im}\,(I_X) \, \mathrm{Im}\,(\omega \, h_\Psi \, h_\Sigma^*) + |g_Z|^2 \, \mathrm{Im}\,(I_Z) \, \mathrm{Im}\,(\omega^2 \, h_\Psi \, h_\Sigma^*) \, .$$

- $I_{X/Z}$ are loop integrals, $\omega = \mathrm{e}^{\frac{2\pi \mathrm{i}}{3}}$.
- Asymmetry vanishes if
  1. $\mathrm{Im}\,(I_X) = \mathrm{Im}\,(I_Z)$, which requires $M_X = M_Z$,
  2. $|g_X| = |g_Z|$ and
  3. $\arg\,(h_\Psi \, h_\Sigma^*) = 0$.
- There is a symmetry corresponding to a $\Delta(27)$ automorphism (not class-inverting) enforcing 1–3:

$$X \leftrightarrow Z \, , \qquad Y \mapsto Y \, , \qquad \Psi \mapsto U \, \Sigma^{\mathcal{C}} \, , \qquad \Sigma \mapsto U \, \Psi^{\mathcal{C}} \, .$$

Just enhances the flavour symmetry to $\Delta(27) \rtimes \mathbb{Z}_2$.

Y decay in the amended model

$$X \leftrightarrow Z\,, \qquad Y \mapsto Y\,, \qquad \Psi \mapsto U\,\Sigma^{\mathcal{C}}\,, \qquad \Sigma \mapsto U\,\Psi^{\mathcal{C}}\,.$$

Just enhances the flavour symmetry to $\Delta(27) \rtimes \mathbb{Z}_2$.

- $\Delta(27) \rtimes \mathbb{Z}_2$ has a physical $\boldsymbol{CP}$ transformation (type II A).
- All coupling phases can be absorbed in field re-definitions.

$\Rightarrow$ $\boldsymbol{CP}$ conserved.

Calculable phases

- Break $\Delta(27) \rtimes \mathbb{Z}_2 \to \Delta(27)$ spontaneously,

$$\mathcal{L} \supset M^2 \left(|X|^2 + |Z|^2\right) + \left[\frac{\mu}{\sqrt{2}} \langle\phi\rangle \left(|X|^2 - |Z|^2\right) + \mathrm{h.\,c.}\right] .$$

  ϕ in non-trivial one-dimensional representation.

- Spontaneous $\boldsymbol{CP}$ violation.

Phase predicted by group theory
$\varepsilon_{Y \to \overline{\Psi}\Psi} \propto \|g_X\|^2 \|h_\Psi\|^2 \,\mathrm{Im}\,(\omega)\, \left[\mathrm{Im}\,(I_X) - \mathrm{Im}\,(I_Z)\right]$

Chen and Mahanthappa (2009)

Building models with spontaneous CP violation

Recipe

- Start with a type II group G.
- Spontaneously break G to type I group $H \subset G$.

$\Rightarrow$ Generically, $\boldsymbol{CP}$ is violated spontaneously.

Conclusion

- Only class-inverting automorphisms define physical CP transformations.
- Basis with real Clebsch–Gordan coefficients
  $\Leftrightarrow$ Bickerstaff–Damhus automorphism.
- Twisted Frobenius–Schur indicator provides a tool to check for real Clebsch–Gordan coefficients and CP transformations.
- Three types of groups:

Type I	Type II A	Type II B
■ No physical CP transformation in a generic setting	■ Basis with real Clebsch–Gordan coefficients ■ CP only constrains phases of couplings	■ No basis with real Clebsch–Gordan coefficients ■ physical CP transformation ■ CP can forbid couplings

Thank You!

Bibliography I

1 R. P. Bickerstaff and T. Damhus, ‘A necessary and sufficient condition for the existence of real coupling coefficients for a finite group’, *Int. J. Quantum Chem.* **27** (1985), 381–391.

2 G. Branco, J.-M. Gerard and W. Grimus, ‘Geometrical T-violation’, *Phys. Lett.* B **136** (1984), 383–386, (inSPIRE).

3 M.-C. Chen, M. Fallbacher, K. T. Mahanthappa, M. Ratz and A. Trautner, ‘CP Violation from Finite Groups’, arXiv: `1402.0507 [hep-ph]`, (inSPIRE).

4 M.-C. Chen and K. T. Mahanthappa, ‘Group theoretical origin of CP violation’, *Phys. Lett.* B **681** (2009), 444–447, arXiv: `0904.1721 [hep-ph]`, (inSPIRE).

5 G. Ecker, W. Grimus and W. Konetschny, ‘Quark Mass Matrices in Left-right Symmetric Gauge Theories’, *Nucl. Phys.* B **191** (1981), 465, (inSPIRE).

6 F. Feruglio, C. Hagedorn and R. Ziegler, ‘Lepton mixing parameters from discrete and CP symmetries’, *JHEP* **2013** (2013), 027, arXiv: `1211.5560 [hep-ph]`, (inSPIRE).

7 *GAP – Groups, Algorithms, and Programming* (2014), The GAP Group, URL: `http://www.gap-system.org`.

8 M. Holthausen, M. Lindner and M. A. Schmidt, ‘CP and discrete flavour symmetries’, *JHEP* **2013** (2013), 122, arXiv: `1211.6953 [hep-ph]`, (inSPIRE).

Bibliography II

9 H. Ishimori, T. Kobayashi, H. Ohki, H. Okada, Y. Shimizu and M. Tanimoto, 'Non-Abelian Discrete Symmetries in Particle Physics', *Progr. Theo. Phys. Suppl.* **183** (2010), 1–163, arXiv: `1003.3552 [hep-th]`, (inSPIRE).

10 N. Kawanaka and H. Matsuyama, 'A twisted version of the Frobenius-Schur indicator and multiplicity-free permutation representations', *Hokkaido Math. J.* **19** (1990), 495–508.