$H \to \gamma\gamma$, Gauge Invariance, and the Hierarchy Problem

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Outline

- Introduction: Regulator dependence in $H \to \gamma\gamma$
- Our analysis strategy
- Results of our calculations
- Implications for BSM physics
- Conclusions

Introduction

- Generally believed that if we're calculating the amplitude for some finite process, we can calculate with any regulator (or no regulator) and we'll get a unique answer.
- Not strictly true.
- A finite calculation can be regulator-dependent if infinities arise in intermediate steps of the calculation; how the infinities cancel can depend on the regulator.
- $H \to \gamma\gamma$ is a finite but regulator-dependent calculation.

Introduction

Quick Intro to $H \to \gamma\gamma$:

- $H \to \gamma\gamma$ does not arise at tree level in SM.
- Arises at 1-loop.
- Main contribution is from $W^{\pm}$ loop; top loop also important.
- Very interesting phenomenologically: sensitive to new heavy particles running in loop.

Introduction

$H \to \gamma\gamma$ calculation:

- Calculate 1-loop contribution with dimensional regularization, get reasonable, gauge-invariant result.
- Calculated in $d = 4$, get dim reg result + extra terms which violate QED Ward identity (Fukuda & Miyamoto 1949).
- Same regulator dependence shows up in $W^{\pm}$ loop, fermion loop, scalar loop.
- Regulator-dependence recognized as ambiguity in calculation; requires physics input to resolve (Jackiw 1999).
- Standard result: want result to respect gauge-invariance, take DR result.

Introduction

- Issue concerns the integral which shows up in $H \to \gamma\gamma$:

$$\int \frac{d^d p}{(2\pi)^d} \frac{4p_\mu p_\nu - g_{\mu\nu}(p^2 - m_f^2)}{(p^2 - m_f^2)^3}$$

- Expression contains two logarithmically divergent pieces with different Lorentz structures:

$$\int \frac{d^d p}{(2\pi)^d} \frac{4p_\mu p_\nu}{(p^2 - m_f^2)^3} \quad \text{and} \quad \int \frac{d^d p}{(2\pi)^d} \frac{-g_{\mu\nu} p^2}{(p^2 - m_f^2)^3}$$

- Log divergence cancels whether using $d = 4 - \epsilon$ or $d = 4$: finite result either way.
- But the finite result is not the same in the two cases.

Introduction

- More precisely, take the integral

$$\int \frac{d^d p}{(2\pi)^d} \frac{4p_\mu p_\nu - g_{\mu\nu}(p^2 - m_f^2)}{(p^2 - m_f^2)^3}$$

- If evaluated in $d = 4$, $4p_\mu p_\nu \to g_{\mu\nu} p^2$, integral $= \frac{i}{(4\pi)^2}(-\frac{g_{\mu\nu}}{2}) \neq 0$.
- If evaluated in Dim Reg, $4p_\mu p_\nu \to 4/(4-\epsilon) g_{\mu\nu} p^2$, and

$$\int \frac{d^{4-\epsilon} p}{(2\pi)^{4-\epsilon}} \frac{4p_\mu p_\nu - g_{\mu\nu}(p^2 - m_f^2)}{(p^2 - m_f^2)^3} = \int \frac{d^{4-\epsilon} p}{(2\pi)^{4-\epsilon}} \frac{g_{\mu\nu}(\frac{\epsilon}{4} p^2 + m_f^2)}{(p^2 - m_f^2)^3}$$

So, in Dim Reg, we get

$$\int \frac{d^{4-\epsilon} p}{(2\pi)^{4-\epsilon}} \frac{4p_\mu p_\nu - g_{\mu\nu}(p^2 - m_f^2)}{(p^2 - m_f^2)^3} = 0$$

- All regulator-dependence in this talk is from this integral.
- Nonzero $d = 4$ term is piece that violates Ward identity in all later calculations.

Strategy

Questions we want to answer:

- How is QED gauge invariance lost in going from $d = 4 - \epsilon$ to $d = 4$?
- What happens if we choose as our physics input, $d = 4$ (i.e., $4p^\mu p^\nu \to p^2 g^{\mu\nu}$ is valid) instead of gauge invariance?

In answering the 2nd question,

- Will *not* abandon gauge invariance.
- Will require that terms in $d = 4$ calculation which violate Ward identity *cancel* when all contributions (SM & BSM) are summed.
- Could rephrase question: With what particle content will $d = 4$ and Dim Reg give same answer?
- Analogy with triangle anomalies in SM: individual diagrams violate Ward identities, but particle content cancels offending terms.
- Unlike triangle anomaly, regulator that preserves Ward identities in $H \to \gamma\gamma$ exists (i.e., dim reg).

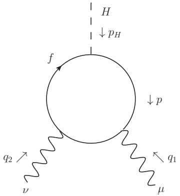
Strategy, cont'd

A few notes about regulators:

- Dim reg gives same result as any gauge-invariant regulator for $H \to \gamma\gamma$.
- $d = 4$ calculation technically equivalent to cutoff regulator; could worry about the regulator dependence of our results.
- Will come back to generality of $d = 4$ result at end.
- Phrased as $d = 4$ vs $d = 4 - \epsilon$ issue, but, another perspective also possible: we're hypothesizing that BSM loops *are nature's regulator* for $H \to \gamma\gamma$.

The Calculation

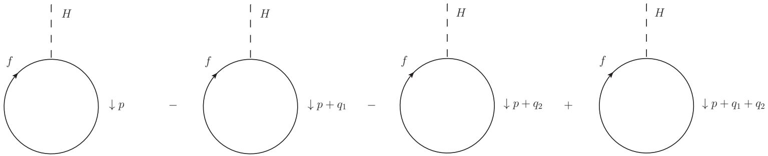
- First, consider fermion loop.



  $+$ diagram with photons interchanged.
- Want to check gauge-inv. of this process: apply Ward ID ($\varepsilon_1^{*\mu}\varepsilon_2^{*\nu} \to q_1^\mu q_2^\nu$), see if 0.
- Remember: For external γ of momentum q attached to fermion loop, applying Ward ID doesn't *identically* give 0.
- Instead, take diagram obtained by removing external photon with momentum q.
- Ward identity gives the difference between this new diagram and same diagram with loop momentum shifted by q.

The Calculation

- So, let's apply the Ward identity to both photons.
- This gives us four terms, each term corresponding to a diagram with both photons removed (tadpoles).
- These terms differ only by loop momentum shifts of q_1 and q_2.
- Obtain $e_f^2 \times$



Note: Will refer to this combination of terms as a "double-shift" of the tadpole.

- Ward ID in $H \to \gamma\gamma$ closely related to shift of corresponding tadpole diagram.

The Calculation

- Now, let's look at the form of that shift. Tadpole diagram is quadratically divergent:

$$i\mathcal{M}^f_{tadpole} = \frac{-4\lambda_f}{\sqrt{2}} \int \frac{d^d p}{(2\pi)^d} \frac{m_f}{p^2 - m_f^2}$$

- The combination of terms that we need is proportional to

$$\int \frac{d^d p}{(2\pi)^d} \left(\frac{1}{p^2 - m^2} - \frac{1}{(p+q_1)^2 - m^2} - \frac{1}{(p+q_2)^2 - m^2} + \frac{1}{(p+q_1+q_2)^2 - m^2} \right)$$
$$= (2) q_1^\mu q_2^\nu \int \frac{d^d p}{(2\pi)^d} \frac{4 p_\mu p_\nu - g_{\mu\nu}(p^2 - m^2)}{(p^2 - m^2)^3}$$

Note: Will return to this expression several times in this talk.

- This is the same regulator-dependent integral we saw before! $(= 0$ in DR, $\neq 0$ in $d = 4)$

Our Calculation

- Above difference of 4 quad.-divergent terms can be written as a diff. of 2 linearly-div. terms which differ only by loop mom. shift.

$$\int \frac{d^d p}{(2\pi)^d} \left[\left(\frac{1}{p^2 - m^2} - \frac{1}{(p+q_1)^2 - m^2} \right) - \left(\frac{1}{(p+q_2)^2 - m^2} - \frac{1}{(p+q_1+q_2)^2 - m^2} \right) \right]$$

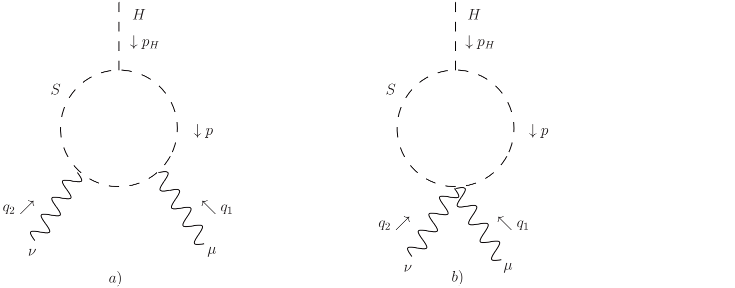
- Linearly-divergent integrals not shift-invariant, but less-than-linearly-divergent ones are (McKeon et al 1982).
- So, why gauge inv. broken in $H \to \gamma\gamma$ when go from $d = 4 - \epsilon$ to $d = 4$?
    - In $d = 4$, expression obtained by applying Ward ID is a difference of two linearly-divergent integrals which differ only by a shift in loop momentum. Nonzero.
    - In $d = 4 - \epsilon$, divergences made less-than-linear, and thus, shift-invariant. Difference between two terms integrates to 0.

Our Calculation

- Note: Any terms less-than-quadratically divergent in tadpole diagrams do not affect $H \to \gamma\gamma$ Ward ID.
- Reason:
  - Any term less-than-quadratically divergent will change, under first loop momentum shift, by an amount which is less-than-linearly divergent.
  - This expression will be invariant under the second loop momentum shift.
  - Hence, change under 2nd loop momentum shift equals 0.
- Only need to know coefficient of quadratic divergence in tadpole diagram.

Our Calculation

- Next, scalar loop:



$$i\mathcal{M}^S_{\mu\nu}\varepsilon_1^{*\mu}\varepsilon_2^{*\nu} = \varepsilon_1^{*\mu}\varepsilon_2^{*\nu} 2\lambda_S v e_S^2 \int \frac{d^d p}{(2\pi)^d} \frac{1}{p^2 - m_S^2} \frac{1}{(p+q_1+q_2)^2 - m_S^2}$$
$$\left[\frac{(2p+q_1)_\mu (2p+2q_1+q_2)_\nu}{(p+q_1)^2 - m_S^2} + \frac{(2p+q_2)_\nu (2p+2q_2+q_1)_\mu}{(p+q_2)^2 - m_S^2} - 2g_{\mu\nu} \right]$$

Our Calculation

- Apply Ward ID:

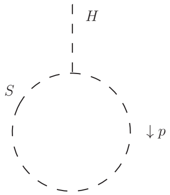
$$i\mathcal{M}^S_{\mu\nu} q_1^\mu q_2^\nu = 2\lambda_S v e_S^2 \int \frac{d^d p}{(2\pi)^d} \frac{1}{p^2 - m_S^2} \frac{1}{(p+q_1+q_2)^2 - m_S^2}$$
$$\times \left[\frac{((p+q_1)^2 - p^2)((p+q_1+q_2)^2 - (p+q_1)^2)}{(p+q_1)^2 - m_S^2} \right.$$
$$\left. + \frac{((p+q_2)^2 - p^2)((p+q_1+q_2)^2 - (p+q_2)^2)}{(p+q_2)^2 - m_S^2} - 2q_1 \cdot q_2 \right]$$

- Simplifies to

$$i\mathcal{M}^S_{\mu\nu} q_1^\mu q_2^\nu = 2\lambda_S v e_S^2 \int \frac{d^d p}{(2\pi)^d} \left[\frac{1}{p^2 - m_S^2} - \frac{1}{(p+q_1)^2 - m_S^2} \right.$$
$$\left. - \frac{1}{(p+q_2)^2 - m_S^2} + \frac{1}{(p+q_1+q_2)^2 - m_S^2} \right]$$

Our Calculation

- Compare to expression for Higgs tadpole diagram via scalar loop:



$$i\mathcal{M}^{S}_{tadpole} = 2\lambda_S v \int \frac{d^d p}{(2\pi)^d} \frac{1}{p^2 - m_S^2}$$

- Expression from applying Ward ID gives $e_S^2 \times$ double-shift of tadpoles, similar to fermion diagram.

Our Calculation

- Can also see this another way. Go back to $H \to \gamma\gamma$ expression:

$$i\mathcal{M}^S_{\mu\nu}\varepsilon_1^{*\mu}\varepsilon_2^{*\nu} = \varepsilon_1^{*\mu}\varepsilon_2^{*\nu} 2\lambda_S v e_S^2 \int \frac{d^d p}{(2\pi)^d} \frac{1}{p^2 - m_S^2} \frac{1}{(p+q_1+q_2)^2 - m_S^2}$$
$$\left[\frac{(2p+q_1)_\mu (2p+2q_1+q_2)_\nu}{(p+q_1)^2 - m_S^2} + \frac{(2p+q_2)_\nu (2p+2q_2+q_1)_\mu}{(p+q_2)^2 - m_S^2} - 2g_{\mu\nu} \right]$$

- We are interested in the on-shell case, $q_1^2 = q_2^2 = 0$, $p_H^2 = m_H^2$.
- Dim Reg calculation is gauge-invariant, so terms which break Ward ID in $d = 4$ must show up in regulator-dependent terms.
- But, if we only want to know difference in going from $d = 4 - \epsilon$ to $d = 4$, we only need to examine log divergent terms.
- Log-divergent terms not dependent on q_1, q_2. Can set $q_1 = q_2 = 0$.
- More precisely, difference between $q_1 = q_2 = p_H = 0$ case and general q_1, q_2, p_H case is finite, regulator-independent.

Our Calculation

- So, simplify $H \to \gamma\gamma$ calculation setting external momenta $= 0$:

$$i\mathcal{M}^S_{\mu\nu}\Big|_{q_{1,2}=0}\varepsilon_1^{*\mu}\varepsilon_2^{*\nu} = \varepsilon_1^{*\mu}\varepsilon_2^{*\nu}4\lambda_S v e_S^2 \int \frac{d^dp}{(2\pi)^d}\frac{4p_\mu p_\nu - g_{\mu\nu}(p^2 - m_S^2)}{(p^2 - m_S^2)^3}$$

- Remembering that

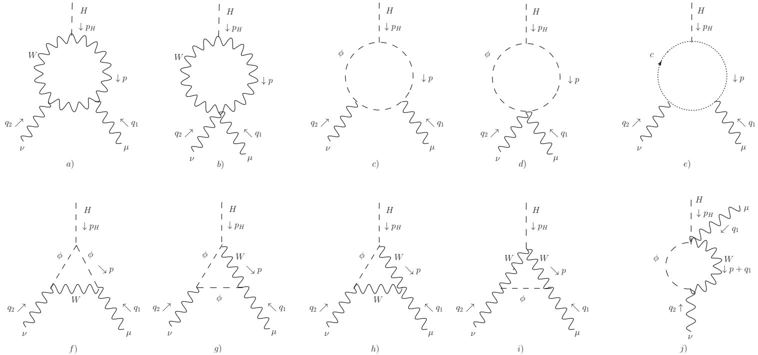
$$\int \frac{d^dp}{(2\pi)^d}\left(\frac{1}{p^2 - m^2} - \frac{1}{(p+q_1)^2 - m^2} - \frac{1}{(p+q_2)^2 - m^2} + \frac{1}{(p+q_1+q_2)^2 - m^2}\right)$$
$$= (2)q_1^\mu q_2^\nu \int \frac{d^dp}{(2\pi)^d}\frac{4p_\mu p_\nu - g_{\mu\nu}(p^2 - m^2)}{(p^2 - m^2)^3}$$

  we see that if we apply Ward ID twice ($\varepsilon_1^{*\mu}\varepsilon_2^{*\nu} \to q_1^\mu q_2^\nu$), get expression in terms of shifts of tadpole terms.

- Again, Ward-ID-violating terms in $d = 4$ $H \to \gamma\gamma$ calc equal to $e_s^2\times$ change of tadpoles under double-shift of loop momentum.

Next, $W^{\pm}$ loop:

- Did calculation in renormalizeable gauge, for general ξ.
- Did not take unitary gauge; all terms in $H \to \gamma\gamma$ finite or log div.
- Need to include all Goldstone, ghost loops.



- Compare to shifts of tadpoles (sum of $W^{\pm}$, Goldstone, and ghost).

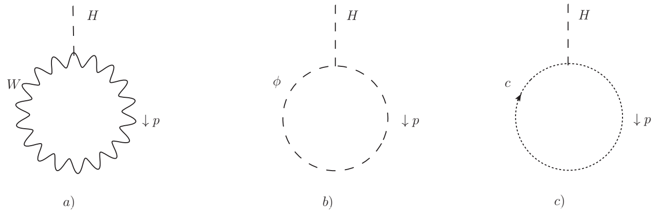
Our Calculation

Strategy for $W^{\pm}$ loop calculation:

- Not using unitary gauge; avoids highly divergent terms which would give momentum-routing ambiguities.
- Like in previous cases, all terms in $H \to \gamma\gamma$ calculation either finite or log divergent.
- Take usual Dim Reg calculation to be gauge-invariant. Terms that violate Ward ID must come from difference in regulators.
- To simplify calculation, take external momenta 0. Difference between this and on-shell case finite, thus regulator-independent.

Our Calculation

- We'll need Higgs tadpole diagrams:



- The amplitudes for these tadpoles are:

$$i\mathcal{M}^{W}_{tadpole} = gM_W \int \frac{d^d p}{(2\pi)^d} \frac{1}{p^2 - M_W^2} \left((4-\epsilon) - \frac{p^2(1-\xi)}{p^2 - \xi M_W^2} \right)$$

$$i\mathcal{M}^{\phi}_{tadpole} = \left(\frac{g m_H^2}{M_W} \right) \int \frac{d^d p}{(2\pi)^d} \left(\frac{1}{2} \right) \frac{1}{(p^2 - \xi M_W^2)}$$

$$i\mathcal{M}^{c}_{tadpole} = gM_W \int \frac{d^d p}{(2\pi)^d} \frac{-\xi}{p^2 - \xi M_W^2}$$

Our Calculation

- $H \to \gamma\gamma$ diagrams with loops only containing Goldstone bosons just like generic scalar shown earlier. Will concentrate on rest of diagrams.
- $W^{\pm}$ and ghost tadpoles sum to

$$i\mathcal{M}^{W+c}_{tadpole} = gM_w \int \frac{d^d p}{(2\pi)^d} \frac{(3-\epsilon)}{(p^2 - M_W^2)}$$

- Rest of $H \to \gamma\gamma$ diagrams sum to

$$i\mathcal{M}^{a,b,e-j}_{\mu\nu} \varepsilon_1^{*\mu} \varepsilon_2^{*\nu}$$
$$= \varepsilon_1^{*\mu} \varepsilon_2^{*\nu} (e^2 g M_W) \int \frac{d^d p}{(2\pi)^d} (6 - 2\epsilon) \frac{4p_\mu p_\nu - g_{\mu\nu}(p^2 - M_W^2)}{(p^2 - M_W^2)^3} + \text{ finite}$$

Note: 2ϵ multiplied by finite integral, will not contribute. Will drop in next slide.

Our Calculation

- Applying Ward ID,

$$i\mathcal{M}^{a,b,e-j}_{\mu\nu} q_1^{\mu} q^{\nu}\Big|_{on-shell}$$

$$= q_1^{*\mu} q_2^{\nu} (e^2 g M_W) \int \frac{d^d p}{(2\pi)^d} (6) \frac{4p_\mu p_\nu - g_{\mu\nu}(p^2 - M_W^2)}{(p^2 - M_W^2)^3} + \text{ finite}$$

- In Dim Reg, this is 0. But, in Dim Reg, integral alone 0. So, finite terms must be 0.
- So, in $d = 4$

$$i\mathcal{M}^{a,b,e-j}_{\mu\nu} q_1^{\mu} q^{\nu}\Big|_{on-shell}$$

$$= q_1^{*\mu} q_2^{\nu} (e^2 g M_W) \int \frac{d^4 p}{(2\pi)^4} (6) \frac{4p_\mu p_\nu - g_{\mu\nu}(p^2 - M_W^2)}{(p^2 - M_W^2)^3} \quad (d = 4)$$

which is just $e^2\times$ a double-shift of

$$i\mathcal{M}^{W+c}_{tadpole} = gM_w \int \frac{d^4 p}{(2\pi)^4} \frac{(3-\epsilon)}{(p^2 - M_W^2)}$$

Our Calculation

To recap:

- For fermion, scalar, and SM $W^{\pm}$ loops, result of applying Ward ID to $H \to \gamma\gamma$ is equal to $e_i^2 \times$ double-shift of corresponding Higgs tadpole diagram ($e_i =$ loop particle charge).
- In $d = 4 - \epsilon$, these terms are 0, in $d = 4$, $\neq 0$.
- Due to difference in behavior under momentum shifts of linearly-divergent vs less-than-linearly-divergent integrals.

Implications for BSM Physics

- What have we learned?
  - In order to get gauge-invariant answer in $H \to \gamma\gamma$, need shift-invariance in diagrams obtained by applying Ward ID (ie, removing photons).
  - We can achieve this with a regulator (like Dim Reg).
  - But it is not the only way! Can also achieve this through the underlying physics.
  - In effect, we can take a lesson from Dim Reg: Take feature of regulator (invariance under loop momentum shifts) and move it into the physics content of the model.
- So, now we try:
  - Take hypothesis that $d = 4$ calculation is valid, gauge-invariance violating terms (SM and BSM) cancel when all contributions are included.
  - Take new physics to be new scalar and fermion loops.
  - For simplicity, we'll assume the SM gauge group (no new vectors).

Implications for BSM physics

- Gauge-invariance-violating terms in $H \to \gamma\gamma$ cancel if quadratic divergences in tadpole diagrams, weighted by loop particle charge^2, sum to 0:

$$e^2 3gM_W + \frac{e^2 g m_H^2}{2M_W} + \sum_{scalars} e_s^2 (2\lambda_S v) - \sum_{fermions} e_f^2 \frac{4\lambda_f m_f}{\sqrt{2}} = 0$$

- Only need to cancel quadratic divergences in tadpoles; less divergent terms do not affect $H \to \gamma\gamma$.
- Tadpole diagrams renormalize Higgs vev v.
- Both m_H and v functions of Higgs potential parameters λ and μ^2.
- Cancelling quadratic divergences in tadpoles equivalent to cancelling quadratic divergences in Higgs self-energy.
- So, would get same expression if wrote down relation needed to cancel quad. div. in Higgs self-energy, but weighted by loop particle charge^2.

Implications for BSM physics

- The cancellation condition that we've written down is not equivalent to the condition to cancel quadratic divergences in m_H, due to weighting by charge^2.
- But it is close!
- Implies that, if we have a model which
  - removes the quad. div. in the Higgs self-energy by the addition of new scalars and fermions,
    and
  - removes these divergences charge-by-charge (ie, all charge 2/3 loops cancel, all charge ± 1 loops cancel, etc.)

  then, in that model, the $d = 4$ $H \to \gamma\gamma$ calculation will be gauge-invariant.
- Quadratic divergences in Higgs self-energy closely related to hierarchy problem, which hopefully LHC will solve. Might simultaneously discover that $d = 4$ $H \to \gamma\gamma$ calculation is, in fact, valid.

Implications for BSM physics

- MSSM cancels quadratic divergences in Higgs self-energy by giving every particle a partner of the same charge.
- This implies $H \to \gamma\gamma$ calculated in $d = 4$ will be gauge-invariant in MSSM.
- Checked explicitly for arbitrary chargino mixing and arbitrary sfermion $L - R$ and flavor mixing from soft breaking terms.
- To perform check, only need to add up all Higgs tadpole contributions, weighted by loop charge^2.

Implications for BSM physics

- Example: Up squark $\tilde{u}$ loop contribution to H_0 tadpole has coefficient

$$e_u^2 \left[\frac{gM_Z}{\cos\theta_W} (I_u \mp e_u \sin^2\theta_W) \cos(\alpha+\beta) + \frac{gm_u^2}{M_W \sin\beta} \sin\alpha \right]$$

- Term $\sim \sin^2\theta_W$ cancels between $\tilde{u}_R$ and $\tilde{u}_L$.
- Last term cancels with quark loop.
- Term $\sim e_u^2 I_u$ cancels when all fermions are summed over; usual anomaly cancellation condition.
- Works!
- Similar for h_0.

Generality of the Results

- $d = 4$ calculation equivalent to a cutoff. What if we had chosen some other regulator?
- Ambiguous integral independent of mass; cancellation condition as derived gives cancellation of ambiguous terms.
- Could have used different value for ambiguous integral. Any nonzero value would have given same cancellation condition.
- Cancellation condition actually gives condition under which $H \to \gamma\gamma$ completely regulator independent. If fulfilled, all regulators will give same result. Gauge invariance automatically enforced by particle content of theory.

Possible extensions

- Expect similar relation for $H \to gg$. Could also consider $H \to Z\gamma, ZZ, W^+W^-$.
- Regulator dependence of finite calculation not unique to $H \to \gamma\gamma$. Similar behavior in photon scattering.
- More loops?
- If cancellation found to hold in nature, interpretation?
  At very least, would indicate that $d = 4$ should not be dismissed in, say, photon scattering.
- Given close connection between $H \to \gamma\gamma$ and Higgs tadpole/self-energy, usual procedure of using dim reg for $H \to \gamma\gamma$ and regulator that retains quadratic divergences for self-energy is somewhat nonintuitive.

Conclusions

- $H \to \gamma\gamma$ has peculiar feature of being finite but regulator-dependent.
- Need gauge-invariant answer: usual procedure is to choose regulator that enforces gauge invariance.
- Shown that it is possible to instead get gauge invariance automatically w/new physics that enforces cancellation of terms that violate Ward ID in $d = 4$.
- If simultaneously insist on gauge invariance and that $d = 4$ result as valid, predicts constraint on BSM particle content.
- Such a constraint is surprisingly easy to fulfill, closely related to diagrams which contribute to quadratic divergences in Higgs self-energy.
- So, not surprising that some models already developed to solve hierarchy problem give sensible results in for $H \to \gamma\gamma$ in $d = 4$.
- If LHC solves the hierarchy problem, very interesting to know if it tells us taking $d = 4$ was OK after all!